Frequency shift and Q of disordered superconducting RF cavities

TTC2022 Aomori, WG-1: Progress of High Q and High Gradient activities

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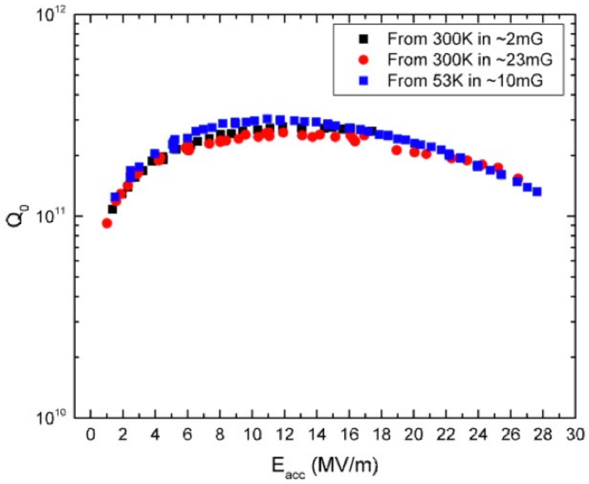
Introduction

- Niobium superconducting radio-frequency (SRF) cavities are high quality electromagnetic resonators.
- Nb SRF cavities with unprecendented quality factors, $Q{\sim}10^{11}$, have been achieved by infusing Nitrogen into the Nb surface [1-3].

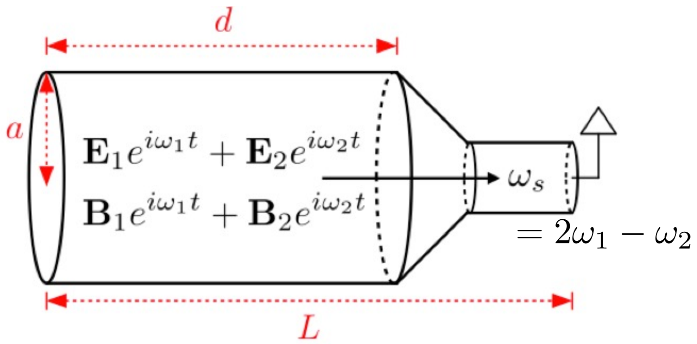


Mechanism of the improvement has not yet fully understood

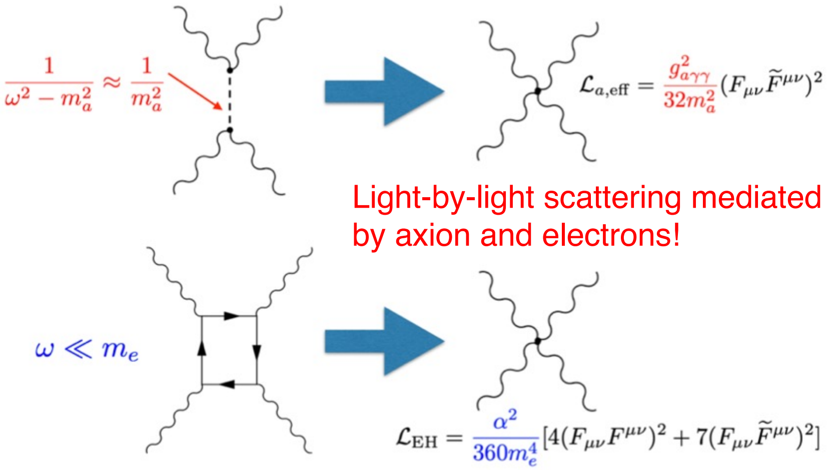
[1] A. Grassellino et al., Supercond. Sci. Technol. **26**, 102001 (2013). [2] A. Romanenko et al., Appl. Phys. Lett. **105**, 234103 (2014). [3] A. Romanenko et al., Phys. Rev. Applied **13**, 034032 (2020).

Introduction

- These high-Q cavities provide a new technology platform for both quantum processors, and quantum sensors for axions including dark matter candidates and the Euler-Heisenberg (EH) term of low-energy QED [4-6].
- The sensitivity to the axion and EH signals depends on the Q of SRF cavities.



Intermodulation due to the nonlinearity in a cavity pumped with two resonance modes

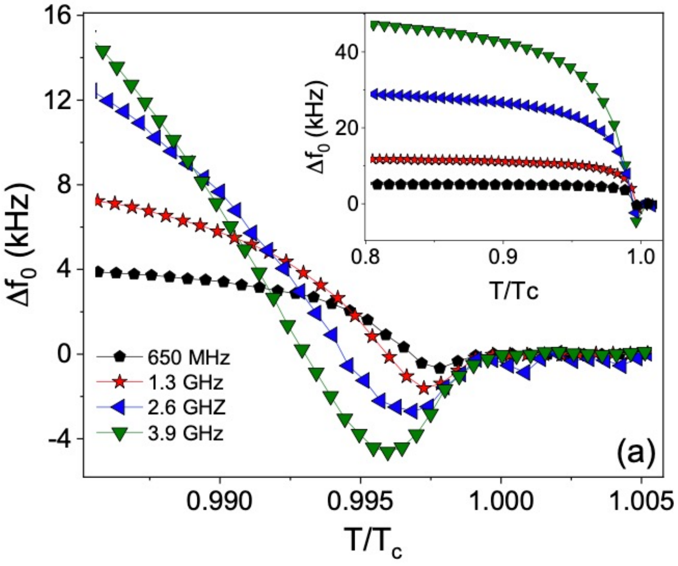


Light-by-light scattering mediated by axion and electrons!

[4] Z. Bogorad et al., Phys. Rev. Lett. **123**, 021801 (2019). [5] Y. Kahn et al., Proc. SPIE **12016**, 1201606 (2022). [6] A. Berlin et al., J. High Energ. Phys. **2020**, 88 (2020).

Introduction

- Bafia et al. measured the frequency shift of N-doped Nb SRF cavities near T_c in detail [7].
- The high-Q SRF cavities have "dip" feature in the frequency shift.



Important to study the N impurity effects on the cavity wall

[7] D. Bafia et al., arXiv:2103.10601.

Introduction

- We have developed numerical methods to calculate the quality factor and frequency shift of N-doped Nb SRF cavities based on the quasiclassical theory of superconductivity [8] and the coupling of charge currents to Maxwell's equations.
- We present theoretical results for the electromagnetic response of N-doped Nb SRF cavities as a function of disorder, temperature, and mode frequency.
- Our theoretical results are in good agreement with experimental results on both the T_c and frequency shift reported in Ref. [7].

[7] D. Bafia et al., arXiv:2103.10601. [8] D. Rainer and J. A. Sauls, "Superconductivity: From Basic Physics to New Developments", ch. 2, pp. 45–78, World Scientific, Singapore (1994).

Formalism

- Conductivity $\sigma = \sigma_1 + i\sigma_2$ calculated based on the quasiclassical theory of superconductivity: [8]

$$\sigma = \frac{\sigma_D}{i\omega\tau}\int_{-\infty}^{\infty}\frac{d\varepsilon}{4\pi i}\left\{\tanh\left(\frac{\varepsilon-\omega/2}{2T}\right)\right.$$
$$\times\frac{-2\pi}{D^{\mathrm{R}}(\varepsilon+\omega/2)+D^{\mathrm{R}}(\varepsilon-\omega/2)+1/\tau}\left[\frac{\varepsilon^2-\omega^2/4+\Delta^2}{D^{\mathrm{R}}(\varepsilon+\omega/2)D^{\mathrm{R}}(\varepsilon-\omega/2)}+1\right]$$
$$+\left[\tanh\left(\frac{\varepsilon+\omega/2}{2T}\right)-\tanh\left(\frac{\varepsilon-\omega/2}{2T}\right)\right]$$
$$\left.\times\frac{-\pi}{D^{\mathrm{R}}(\varepsilon+\omega/2)+D^{\mathrm{A}}(\varepsilon-\omega/2)+1/\tau}\left[\frac{\varepsilon^2-\omega^2/4+\Delta^2}{D^{\mathrm{R}}(\varepsilon+\omega/2)D^{\mathrm{A}}(\varepsilon-\omega/2)}+1\right]\right\},$$

$$D^{\mathrm{R,A}}(\varepsilon)\equiv\sqrt{\Delta^2-(\varepsilon\pm i0^+)^2},$$

σ_D: Drude conductivity.

[8] D. Rainer and J. A. Sauls, "Superconductivity: From Basic Physics to New Developments", ch. 2, pp. 45–78, World Scientific, Singapore (1994).

Formalism

- Surface impedance $Z_s = R_S + iX_s$ obtained by solving Maxwell's equations on interface between vacuum and superconductor:

$$\frac{R_s}{R_n} = \frac{\sigma_{n1}^{1/2}}{(\sigma_1^2+\sigma_2^2)^{1/4}}\left[\cos\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right) - \sin\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right)\right],$$
$$\frac{X_s}{R_n} = \frac{\sigma_{n1}^{1/2}}{(\sigma_1^2+\sigma_2^2)^{1/4}}\left[\cos\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right) + \sin\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right)\right].$$

Normal-state resistance R_n, reactance X_n, and conductivity σ_{n1}:

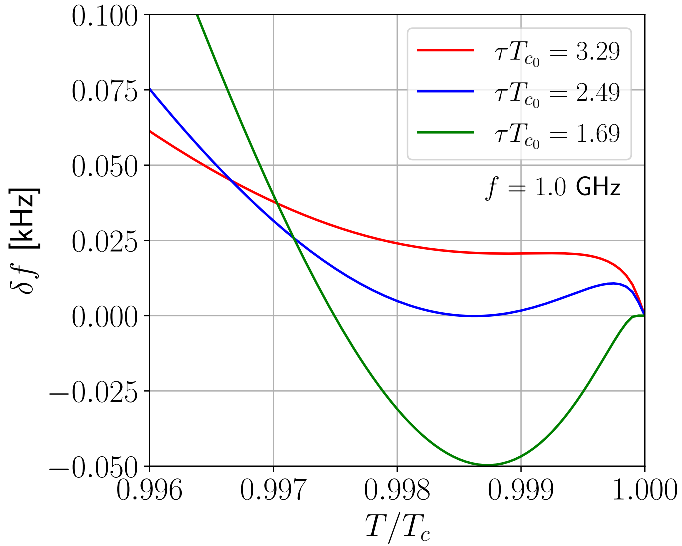
$$R_n = X_n = \frac{4\pi}{\omega_p c}\sqrt{\frac{\pi f}{\tau}}, \quad \sigma_{n1} = \frac{\sigma_D}{1+(\omega\tau)^2}, \quad \omega_p\text{: plasma frequency.}$$

- Quality factor Q and frequency shift δf of the SRF cavities calculated from Maxwell's equations in a hollow cavity based on the Slater method: [9]

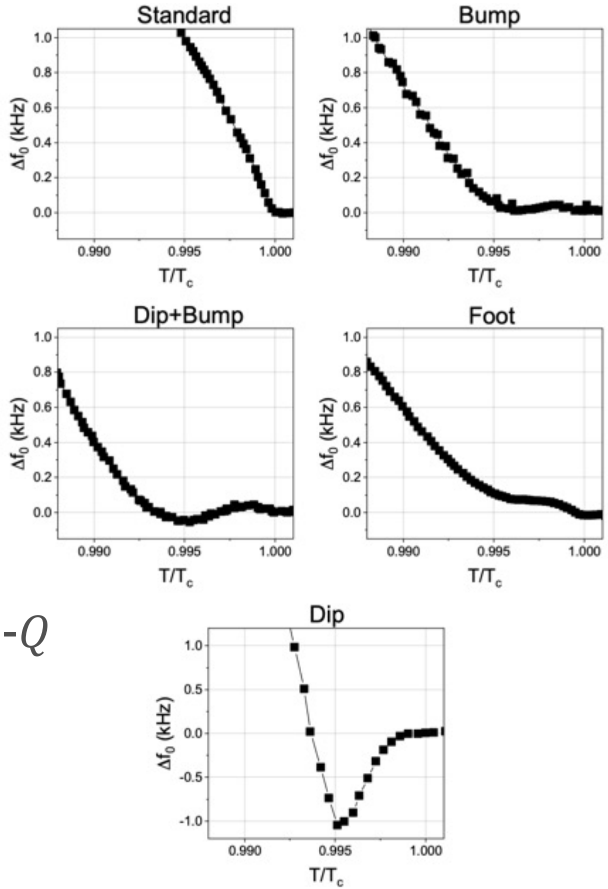
$$Q = \frac{G}{R_s}, \quad \delta f = \frac{f}{2G}(X_n - X_s), \quad G = \frac{8\pi^2 f}{c^2}\int_V \mathbf{H}^2 dv \Big/ \int_S \mathbf{H}^2 da.$$

[9] J. C. Slater, Rev. Mod. Phys. **18**, 441 (1946).

Theoretical Calculation of Frequency Shift

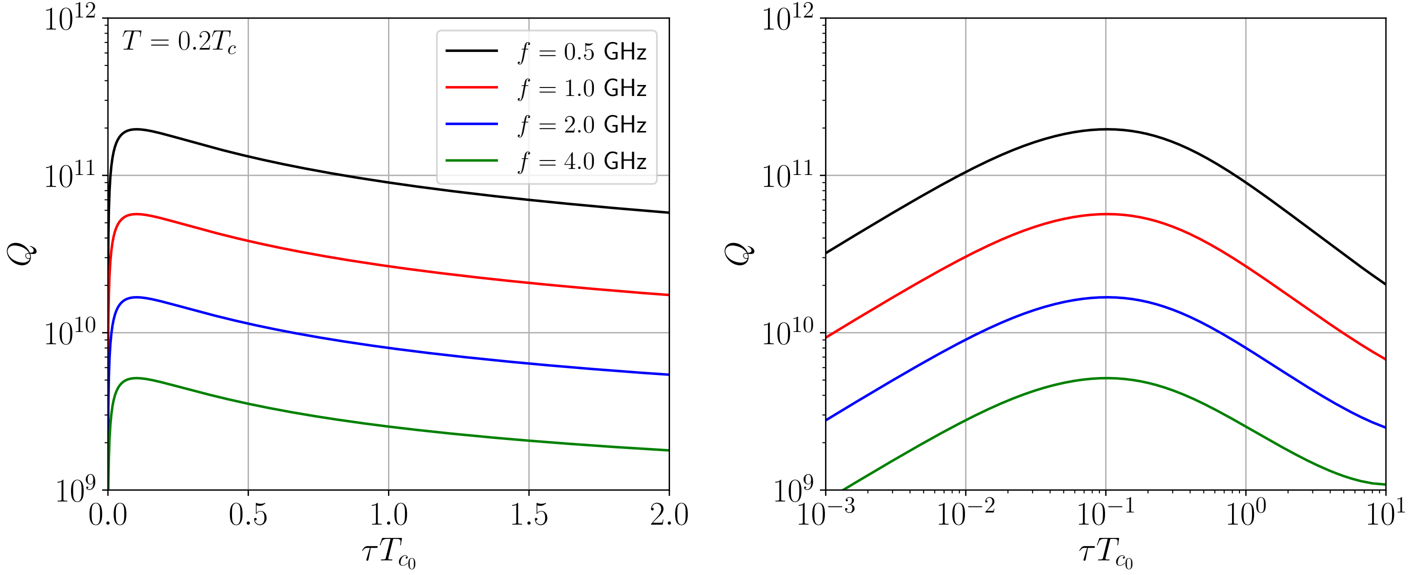


[7] D. Bafia et al., arXiv:2103.10601.



- The green line has the "dip" feature as seen in the high-Q cavities.
- The red and blue lines are the same behavior as the experimental data called the "foot" and "bump" feature, respectively [7].

Theoretical Calculation of Quality Factor



- The quality factor in cavities with intermediate disorder is the largest.
- It becomes rather small in the too dirty cavities due to the pair breaking.

Comparison & Analysis of N-doped Nb SRF Cavities

- Bafia et al. measured the frequency shift of Nb SRF cavities near T_c in detail [7].
- The weak coupling theory cannot fit well with the peak position and spread in the dip of the frequency shift in the experimental data.
- The transition temperature varies depending on where it is measured.
- We consider this T_c spread in our calculations.

[7] D. Bafia et al., arXiv:2103.10601.

Superconducting Gap with Spread in T_c

- Gap energy with spread in T_c: [11]

$$\Delta(T) = \Delta_0 \sqrt{\int_{-\infty}^{\infty} dT_c\, \rho(T_c) \tilde{\Delta}^2(T, T_c)}, \quad \Delta_0 = \pi e^{-\gamma} T_c^{\mathrm{ave}},$$

$$\tilde{\Delta}(T, T_c) = \tanh\left(\frac{\pi T_c}{\Delta_0} \sqrt{\frac{8}{7\zeta(3)} \frac{T_c - T}{T}} \right) \Theta(T_c - T).$$

- Gaussian distribution of T_c: [11]

$$\rho(T_c) = \frac{1}{\sqrt{2\pi} T_c^{\mathrm{SD}}} \exp\left[-\frac{1}{2} \left(\frac{T_c - T_c^{\mathrm{ave}}}{T_c^{\mathrm{SD}}} \right)^2 \right].$$

Average of T_c: $T_c^{\mathrm{ave}} = (T_c^{\mathrm{max}} - T_c^{\mathrm{min}})/2.$

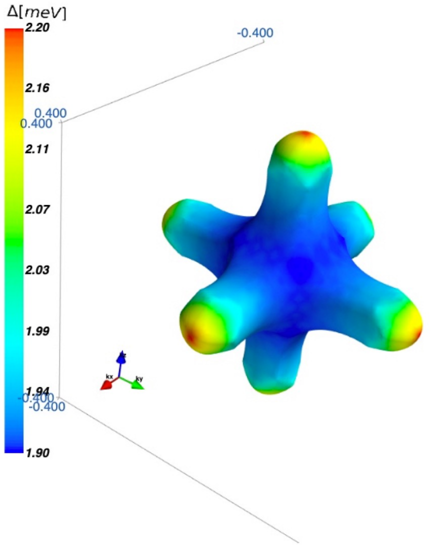
Standard deviation of T_c: $T_c^{\mathrm{SD}} = (T_c^{\mathrm{ave}} - T_c^{\mathrm{min}})/3.$

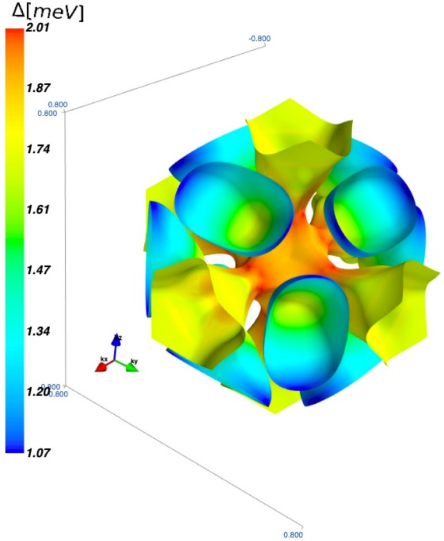
T_c^{max} and T_c^{min} are fitting parameters

[11] HU, M. Zarea, and J. A. Sauls, arXiv:2207.14236.

Spread in T_c and Inhomogeneity of Impurities

- T_c equation: $$\ln \frac{T_{c_0}}{T_c} = A \sum_{n=0}^{\infty} \left(\frac{1}{n+\frac{1}{2}} - \frac{1}{n+\frac{1}{2}+\frac{1}{2}\frac{1/\tau}{2\pi T_c}} \right), \quad A \equiv \frac{\langle |\Delta(\mathbf{p})|^2 \rangle - |\langle \Delta(\mathbf{p}) \rangle|^2}{\langle |\Delta(\mathbf{p})|^2 \rangle}.$$
- Transition temperature for pure Nb: $T_{c_0} = 9.33$ K.
- Gap anisotropy factor: $A = 0.037$.





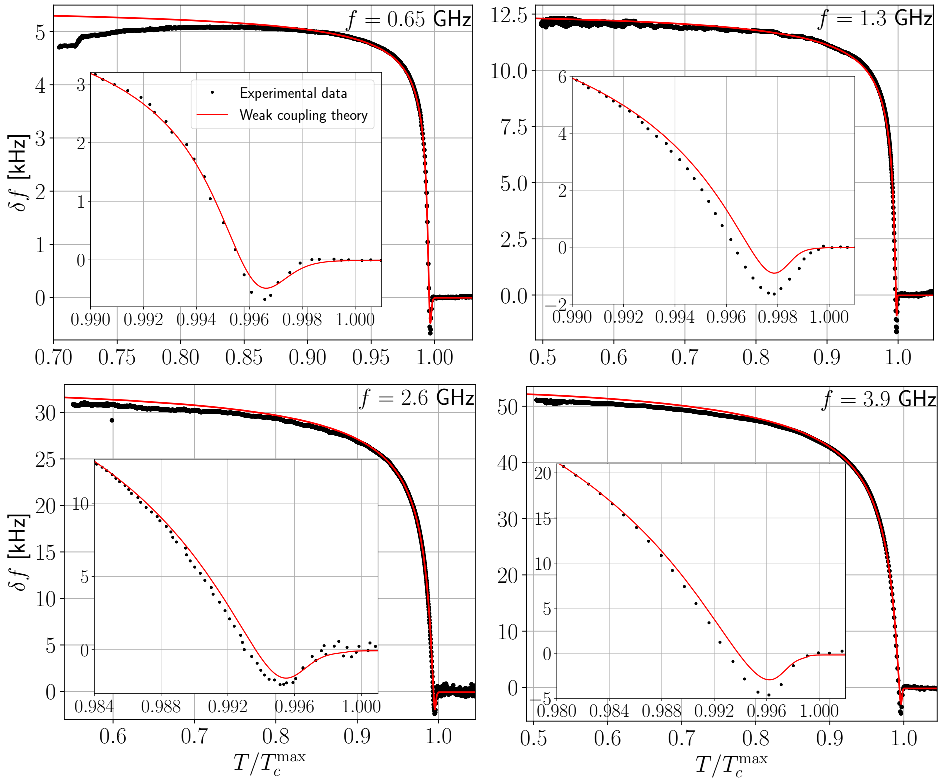
This T_c spread comes from inhomogeneity of impurities.

These parameters are obtained from the LDA calculation [12].

[12] M. Zarea, HU, and J. A. Sauls, arXiv:2201.07403.

Comparison & Analysis of N-doped Nb SRF Cavities

- Frequency shift of the N-doped Nb SRF cavities with the different frequency: [7]

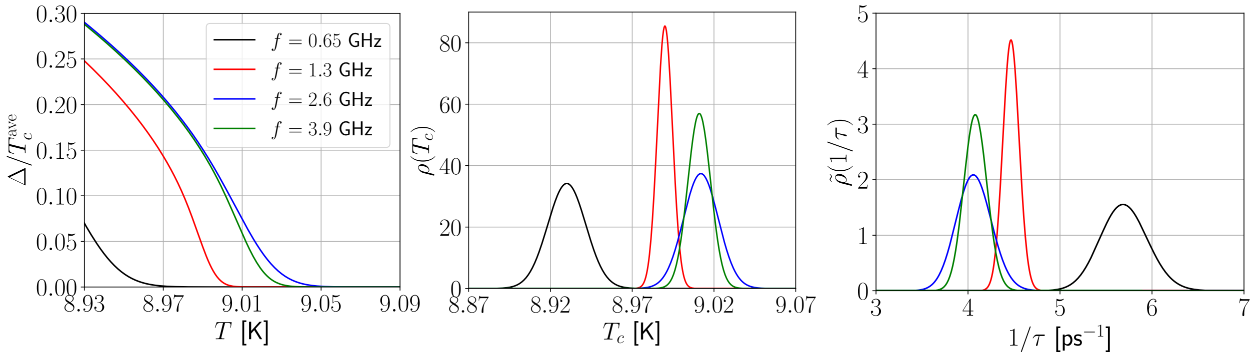


Our theoretical lines are fitted well with the experimental data!

[7] D. Bafia et al., arXiv:2103.10601.

Comparison & Analysis of N-doped Nb SRF Cavities

- Gap energy near T_c and Gaussian distributions of T_c and $1/\tau$:



- Used T_c^{max}, T_c^{min}, R_n, and the corresponding experimental data: [7]

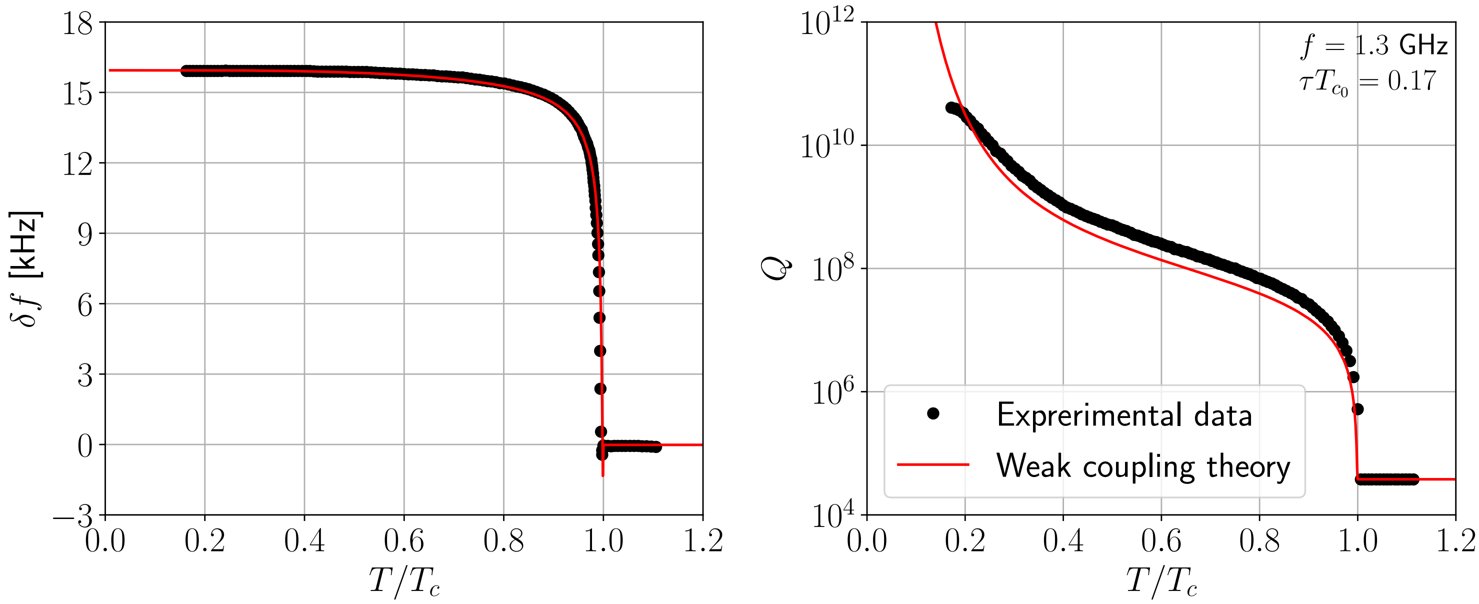
	Theory				Experiment			
f [GHz]	0.65	1.3	2.6	3.9	0.65	1.3	2.6	3.9
$T_c^{\max}$ [K]	8.965	9.004	9.044	9.032	9.005	8.907	9.081	9.165
$T_c^{\min}$ [K]	8.895	8.976	8.980	8.990	8.975	8.87	9.041	9.15
R_n [mΩ]	4.471	5.601	7.554	9.272	4.364	5.425	6.95	8.93

[7] D. Bafia et al., arXiv:2103.10601.

Our theory is in good agreement with the experimental data!

Comparison & Analysis of N-doped Nb SRF Cavities

- Quality factor and frequency shift of the Nb SRF cavity: [7]



The calculation of Q is not perfect but in reasonable agreement with the experimental data.

[7] D. Bafia et al., arXiv:2103.10601.

Summary

- We showed that the quality factor has a peak of upper convexity as a function of the quasiparticle-impurity scattering rate, with the largest Q in cavities with intermediate disorder.

- We presented theoretical results for the effects of inhomogeneous disorder on the transition temperature and frequency shift of SRF cavities and our calculations are in good agreement with the experimental results [7].

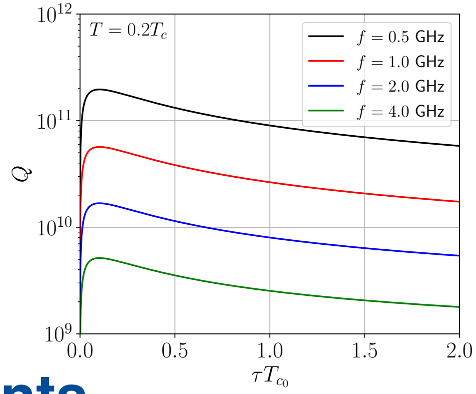
- We provide a new tool for characterization of high-Q SRF cavities.

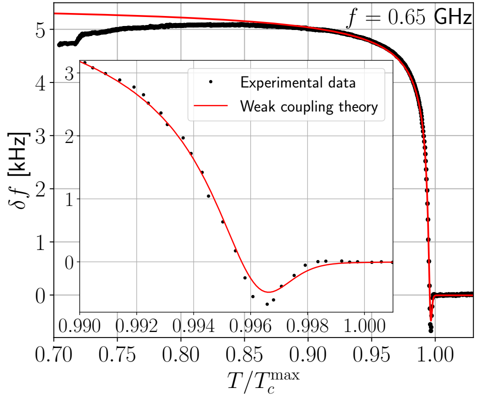
We thank D. Bafia, A. Grassellino, A. Romanenko, and J. Zasadzinski for many discussions on their experimental work on SRF cavities.

[7] D. Bafia et al., arXiv:2103.10601.

Main Message

- The quality factor has a peak of upper convexity as a function of the impurity scattering rate, with the largest Q in cavities with intermediate disorder.
- The effects of inhomogeneous disorder on T_c are very important for the frequency shift of the SRF cavities near T_c.



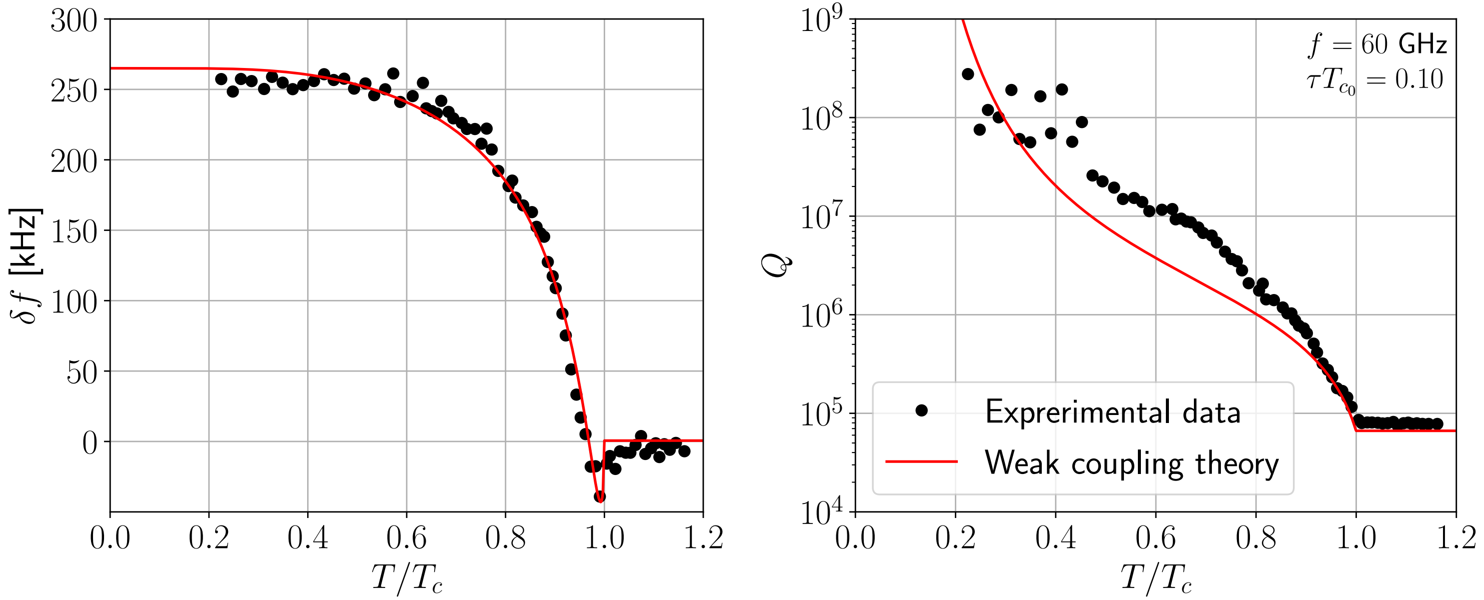


Taking Points

- Why is Q the largest at intermediate levels of disorder?
- Why do the Nb SRF cavities with intermediate levels of disorder have the dip feature in the frequency shift?

Comparison & Analysis of a Nb Sample in a Cooper Cavity

- Frequency shift and quality factor of a Nb sample in a cooper cavity: [10]



The comparison for Q is not perfect, but the experimental data does have significant scatter. That for δf is much better.

[10] O. Klein et al., Phys. Rev. B **50**, 6307 (1994).

Frequency Shift Anomaly Near T_c

To understand the dip in the frequency shift, we express the conductivity as [11]

$$\sigma_1 = \sigma_{1n} + \delta\sigma_1, \quad \sigma_2 = \sigma_{2n} + \delta\sigma_2.$$

$\delta\sigma_1$ and $\delta\sigma_2$ are the small deviations from σ_{1n} and σ_{2n}, respectively.

Assuming $\sigma_{1n} \gg \sigma_{2n}$, we obtain the frequency shift near T_c as

$$\delta f = \frac{fR_n}{4G\sigma_{1n}}(\delta\sigma_1 - \delta\sigma_2).$$

$\delta\sigma_1 - \delta\sigma_2 < 0$ is now satisfied since $\delta\sigma_2$ is larger in dirty superconductors, and then the frequency shift becomes negative near T_c.

[11] HU, M. Zarea, and J. A. Sauls, arXiv:2207.14236.